

On the Need for Insurance Analysts and Actuaries to reconsider Keynes's Logical Theory of Probability, Part I: Russell's Refutation of Ramsey's Critique of Keynes's Theory in 1922

Michael Emmett Brady ^{a,*}

^a Adjunct Lecturer, California State University, Dominguez Hills, College of Business Administration and Public Policy, Department of Operations Management

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ABSTRACT

A historical myth has been created in the 20th and 21st centuries that F P Ramsey uncovered major technical and logical flaws in Keynes's logical theory of Probability that was presented in his 1921 *A Treatise on Probability*. It has been accepted in academia that Ramsey showed in 1922, in his book review in the January issue of *Cambridge Magazine*, and then again with greater force in his 1926 paper, titled "Truth and Probability", that Keynes's relational propositional logic contained serious logical mistakes that made it impossible to base probability and statistics on.

However, it has been completely overlooked for 100 years that it took Bertrand Russell only one small footnote to refute all of Ramsey's claims, that supposedly demonstrated the logical errors in Keynes's relational propositional logic as presented by Keynes on pp.4-6 and pp.53-56 of the *A Treatise on Probability*, with one small counter example that refutes Ramsey's entire argument. Russell's refutation occurs in his July, 1922 review, published in the *Mathematical Gazette* on pp.119-125. Russell's refutation takes place on page 120 in a footnote.

This explains why Ramsey's work on probability and statistics is never mentioned by Russell in his monumental 1948 *Human Knowledge: Its scope and limits*. The reason is that Ramsey's positive contributions to his own subjective theory of probability are severely marred because these positive contributions are hopelessly intertwined with a series of false claims made about Keynes's logical theory of probability. These false claims are identical in nature to the false claims contained in Ramsey's first critique of Keynes in 1922.

It is thus imperative that insurance analysts seriously think about reconsidering Keynes's theory as it is built upon a Boolean algebra and logic that is the foundation of many fields today such as artificial intelligence and computer languages.

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*Corresponding Author:
mandmbrady@juno.com

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Section 1. Introduction

The paper will be organized in the following manner .Section Two will present Ramsey's four examples that he claimed demonstrated that Keynes's relational propositional logic was severely flawed .Section Three will present Russell's simple and straight forward refutation of Ramsey's major point in both of his reviews in 1922 and 1926.Russell's demonstration of the errors of Ramsey's claims was done in the same manner that Adam Smith dealt with Jeremy Bentham and Sir James Steuart in later editions of *The Theory of Moral Sentiments* and in *The Wealth of Nations* ,respectively .Russell never mentions Ramsey's name or cites from the article that Ramsey published in the *Cambridge Magazine*. However, Russell's counter example makes it very clear beyond a doubt that Russell had destroyed Ramsey's examples completely and totally. Ramsey's critique ,then ,in no way showed any flaws or errors in the framework of Keynes's logical approach to probability and statistics.

Section 2. Ramsey's Four examples

Ramsey's examples all involve the same type of claim .He claims that Keynes's theory is highly restricted because it can handle only two propositions at a time .Ramsey also claimed that Keynes's logic was based on a choice of any pair of propositions that were not contradictory. Ramsey then claimed that it could be shown by counter examples that Keynes's definition ,which he stated was contained in Keynes's axiom I, led to probability assessments between propositions that were irrelevant.

Consider the following examples given by Ramsey.

Ramsey gave one example in his 1922 *Cambridge Magazine* review of Keynes's *A Treatise on Probability* and three in his 1926 "Truth and Probability" review, published in 1931 and republished in Kyburg and Smokler,1980.Ramsey claimed that these examples refuted Keynes's logical theory of probability. They do not refute Keynes's theory because all of them are badly flawed ,either in part or whole.

Consider the first paragraph of Ramsey's 1922 paper:

"Mr. Keynes takes probabilities or probability relations as indefinable and says that if q has to p the probability relation of degree a , then knowledge of p justifies rational belief of degree a in q ."(Ramsey,1922,p.3).

This is an incomplete statement of Keynes' position ,since Ramsey never states that the propositions p and q must be related and /or associated to one another in such a way as to form an argument. Keynes's argument form requires that one proposition(the premises denoted by h) provides relevant evidence for the second proposition (the conclusion denoted by a).Further ,there can be more than one premise and/or more than one conclusion. It is not restricted to one h proposition and one a proposition as asserted by Ramsey without any citation to any page in the *A Treatise on Probability*. Keynes made it very clear that his logic applies to sets of propositions .

Consider the third paragraph in Ramsey's 1922 review:

"First, he (author's note-Keynes) thinks that between any two non-self-contradictory propositions there holds a probability relation (Axiom I), for example between 'My carpet is

blue' and 'Napoleon was a great general'; it is easily seen that it leads to contradictions to assign the probability $1/2$ to such cases, and Mr. Keynes would conclude that the probability is not numerical. But it would seem that in such cases there is no probability; that, for a logical relation, other than a truth function, to hold between two propositions, there must be some connection between them. If this be so, there is no such probability as the probability that 'my carpet is blue' given only that 'Napoleon was a great general', and there is therefore no question of assigning a numerical value.”(Ramsey, 1922, pp.3-4).

Nowhere in anything written by Keynes in his lifetime does he state that “... that between any two non-self-contradictory propositions there holds a probability relation (Axiom I)...” Again, Ramsey makes the same mistake that he did in his opening paragraph-Ramsey ignores the precise argument form that the propositions must have in order to satisfy Keynes’s definition of argument form on pages 4-6 of the TP.

The claim made by Ramsey has nothing to do with Keynes’s use of propositions, which must be stated in the form of an argument (Keynes, 1921, p.4)-one proposition must contain relevant evidence while the second proposition must be a conclusion with respect to the proposition containing the relevant evidence. Only then is a relation of logical probability present. Nowhere at any place in his *A Treatise on Probability* or any other work written in Keynes’s lifetime did Keynes state “...that between any two non-self-contradictory propositions there holds a probability relation (Axiom I)...”(Ramsey, 1922, p.3).

Further, there is no such Axiom I in Keynes’s *A Treatise on Probability*. Therefore, Ramsey’s “... 'My carpet is blue' and 'Napoleon was a great general...’” example is an oxymoron because his two propositions do not form an argument form (see Keynes, 1921, p.4). Ramsey’s example is simply gobbledygook.

Nor would Keynes ever claim that “Mr. Keynes would conclude that the probability is not numerical.” (Ramsey, 1922, p.3).

Keynes would conclude that the probability is not defined.

Let us now consider Ramsey’s three examples in 1926 :

“Besides this view is really rather paradoxical; for any believer in induction must admit that between 'This is red' as conclusion and 'This is round', together with a billion propositions of the form 'a is round and red' as evidence, there is a finite probability relation; and it is hard to suppose that as we accumulate instances there is suddenly a point, say after 233 instances, at which the probability becomes finite and so comparable with some numerical relations.”(Ramsey, 1980, p.28; Kyburg and Smokler, 1980).

This example directly violates the argument form put forth by Keynes on pp.4-6 of the TP, since there is no connection between “This is red” and “This is round” because these two propositions are not related, so that neither proposition provides any information or evidence with regards to the other. There is no conditional probability for (this is red/given that that is round). Again, given the fact that the example has nothing whatsoever to do with anything that is in Keynes’s book, one can only conclude that it is again gobbledygook. Note also Ramsey’s attempt to introduce a joint probability (“and”) that has nothing to do with Keynes’s theory based on conditional probability.

This example was preceded by the following example, which can be fixed up only if both separate parts are combined :

“If, on the other hand, we take the simplest possible pairs of propositions such as 'This is red' and 'That is blue' or 'This is red' and 'That is red', whose logical relations should surely be easiest to see, no one, I think, pretends to be sure what is the probability relation which connects them. “(Ramsey,1980,p.28;In Kyburg and Smokler ,1980).

Ramsey's 'This is red' and 'That is blue' example suffers from the same problem as his 'My carpet is blue' and 'Napoleon was a great general' example ,as well as the example above, that 'this is red and 'this is round'. Neither of the two propositions is related in each example. His “or” part, that 'This is red' and 'That is red' ,is connected .All probabilities are conditional for Keynes, so we can rewrite this as the conditional probability that ‘this is red’ ,given that ‘ that is red’ .We can write that ‘This is red’ is the a proposition(conclusion) and ‘ that is red’ is the h proposition(available evidence) $\geq 1/2$,as the only other possible conclusion, not red ,is not supported by any evidence. In Brady(2004a ,b), it was shown that Ramsey's “or” can be replaced with an “and” ,leading to the conclusion that the conditional probability (this is red, given that is red) $>$ the conditional probability (this is red, given that is blue).

Consider Ramsey's final example, a coin example :

“It is true that about some particular cases there is agreement, but these somehow paradoxically are always immensely complicated; we all agree that the probability of a coin coming down heads is $1/2$, but we can none of us say exactly what is the evidence which forms the other term for the probability relation about which we are then judging.”(Ramsey,1980,p.28.In Kyburg and Smokler,1980).

The problem here is that Ramsey is overlooking the question of whether the coin is a fair coin or is not a fair coin. Ramsey hides the evidence by presenting the problem as being one of calculating a marginal probability, so that we have the $P(H)=1/2$,as opposed to $P(H/the\ coin\ is\ fair)=1/2$,which is a conditional probability.

Therefore , we replace Ramsey's “...we all agree that the probability of a coin coming down heads is $1/2$.” by “...we all agree that the probability of a FAIR coin coming down heads is $1/2$.”,where the evidence has been subsumed in the word “fair”.

Ramsey's claim ,that we all agree that a coin coming up heads is $1/2$,is false because the probability of an UNFAIR coin coming up heads IS NOT $1/2$.It is correct to conclude that only a FAIR coin coming up heads is $1/2$.

Section 3. Russell's logical refutation of the foundation of all of Ramsey's claims

Consider the following from Russell's 1922 ,July, review of Keynes's A Treatise on Probability:

“The above form is a necessary but not a sufficient condition of equiprobability. In order to arrive at a sufficient condition, it is first necessary to define irrelevance. It would be natural to say: h_1 is irrelevant to x on evidence h if $x/h_1h = x/h$, i.e., if the addition of h_1 to our data makes no difference to the probability of x . But h_1 may consist of two parts, one of which increases the probability of x while the other diminishes it. To exclude this possibility, we define: h_1 is

irrelevant to x/h if there is no proposition h_1 , inferable from h_1/h but not from h , such that $x/h/h \neq x/h$.

A proposition is defined as relevant when it is not irrelevant*.”(Russell,1922,p.120).

Russell’s * footnote provides the following refutation of all of Ramsey’s examples by showing that the structure of Ramsey’s examples are all prevented from occurring by Keynes’s relevance -irrelevance logic put forth on pp.52-56 of Keynes’s book:

“* I do not know whether Mr. Keynes has considered and rejected a definition of irrelevance which, prima facie, would be simpler than his. He does not state definitely whether every pair of propositions has some probability-relation, but I think he does not hold this view. I think he would say, e.g., that there is no probability-relation between the propositions ' $2+2=4$ ' and ' Napoleon disliked poodles.' If so, it would seem natural to define h as irrelevant to a when a/h does not exist.”(Russell,1922,p.120).

Of course, Ramsey’s main example in his January 1922 review involves Napoleon. It is not a coincidence that Russell has chosen his second proposition to involve Napoleon, just as Ramsey’s second proposition involved Napoleon. Now Keynes had made it clear on pp.4-6 of the TP that propositions that were not associated ,related ,similar, or connected were excluded from use in his relational propositional logic.

Keynes’s exposition of his relevance -irrelevance logic in chapter 4 of the TP confirms Russell’s conjecture that “...He does not state definitely whether every pair of propositions has some probability-relation, but I think he does not hold this view.”

C. Misak (See Misak,2020,p.114) simply accepted the claim promoted by Ramsey ,and later by Braithwaite ,as being a true reflection of Keynes’s approach when it is completely false.

Section 4. Conclusions

Russell’s simple counter example should have led to the complete rejection of Ramsey’s reviews. Instead ,Ramsey’s gobbledygook was accepted as the gospel truth because ,after all, he was the 18 year teenage boy genius ,wasn’t he?

Russell’s example has never been cited or used as a counter example showing the foolishness of Ramsey’s claims in the 100 years since Russell published his paper in July,1922.Russell’s example should have been the end of any serious consideration of Ramsey ‘s nonsense about Keynes’s logical theory of probability. It is a real mystery why 1000’s of academicians for over 100 years have come to believe the utter nonsense that Ramsey demolished ,destroyed and decimated Keynes’s theory.

It is thus incumbent on analysts and actuaries working in the insurance industry to take another look at what Keynes did,as Keynes shows how statistics (Part V-pp.395-425), induction and analogy(Part III-pp.233-238,253-258) can be built on a framework that uses Boolean logic and algebra as well as probability(chapters I and II;X-XVII)

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