Can the Probability of Being in a Crisis Phase Detect the Bursting of Speculative Bubbles?

Abdessamad Ouchen a,*

* PhD, National School of Business and Management, Sidi Mohamed Ben Abdellah University, Morocco

ABSTRACT

The choice of our topic is due to the recurrence of financial crises. The world today is deeply unstable and subject to uncertainties and big surprises. Finance is known for two regimes: state of stability and state of crisis. Therefore, in order to understand the cyclical asymmetries in the series of yields of the main indices of the world, one has to resort to non-linear specifications that distinguish between upswings and downturns. We estimated a switching model in both states and with a specification autoregressive of order 1, the monthly first difference of the S&P 500 during the period running from December 1999 to December 2015. This model allowed us to confirm the existence of two regimes distinct on the Wall Street Stock Exchange, namely the state of crisis and that of stability. It allowed the detection of three bubbles: the dot.com bubble (1998-2000), the real estate bubble (1995-2006) and the Chinese financial bubble (2014-2015). Indeed, the probability of being in crisis phase (probability smoothing) is greater than 0.6 after the crisis of TMT (2000-2001), after the financial crisis between 2007 and 2008, after the European debt crisis in 2010 and after the Chinese financial crisis in 2015.
1. Introduction

During the last twenty years, no fewer than ten financial crises: the collapse of Barings Bank in 1995, the Mexican crisis between 1994 and 1995, the Thai crisis and that of all the countries of Southeast Asia between 1997 and 1998, the Russian crisis in 1998, the near collapse of LTCM (Long-term capital management) in 1998, bursting the Internet bubble and TMT (Telecommunications Media Technology) between 2000 and 2001, the Argentine crisis in 2001-2002, the financial crisis between 2007 and 2008, the sovereign debt crisis in the euro area began at the end of 2009 and the bursting of the Chinese financial bubble in 2015. This litany of financial crises shows indeed that our universe became turbulent and marked by the unthinkable. The choice of our topic is due to the recurrence of financial crises. The world today is deeply unstable and subject to uncertainties and big surprises. Finance is known for two regimes: state of stability and state of crisis. Therefore, in order to understand the cyclical asymmetries in the series of yields of the main indices of the world, one has to resort to non-linear specifications that distinguish between upswings and downturns. Nonlinear models include, among others, nonlinear error correction model, the autoregressive TAR threshold (Tiao and Tsay 1994), the SETAR model (Tersvirta and Anderson 1992), the Markov switching model of Hamilton (1989)... Having enjoyed success in the analysis of quarterly gross domestic product of the United States, the Markov switching model constitutes adequate econometric tool to be taken into consideration for cyclical asymmetries in the series of our variable, namely: the monthly first difference of the S&P 500, that is to say the nonlinearity in this series. This is an approach which can identify and detect turning points in both peaceful and crisis phases of financial time series. Hamilton (1989) shows that the first difference of the observed financial series, which are generally non-stationary in level, following a nonlinear stationary process.

In this article, we focus on the theoretical foundations of financial shocks and speculative bubbles and their detection by the Markov switching model. It consists of four sections. The first section is fixed about data example and time series models. The second section is devoted to a presentation of the speculative bubble theory. The third section is reserved for the presentation of the Markov regime switching model. The last section focuses on the detection of financial crises using the Markov regime switching model.
2. Data of the Study and Time Series Models

The observations of the variable of our interest, that is to say, the monthly observations of the first difference of the S & P 500 that range from January 2000 until December 2015, are a stationary financial time series or a stationary stochastic process. We adopted the first difference in our series because false regressions can occur if the time series is not stationary. The financial series are not stationary in level, while they are stationary in first differences.

A stochastic process is said to be stationary if its mean and variance are constant over time and if the covariance between two periods of time depends only on the difference between the two periods and not on when the covariance is calculated. While the non-stationary time series is characterized by a variable time average or a variance fluctuating in time or both. Therefore, before beginning the analysis of a stochastic process, we have to examine its stationarity through the use of unit root tests, such as: Dickey-Fuller test, Augmented Dickey-Fuller test (ADF test), Phillips-Perron test, and KPSS (Kwiatkowski, Phillips, Schmidt and Shin) test.

![Figure 1](image)

**Figure 1**
S&P500 and DS&P500 during the period 2000-2015

On the basis of these graphs, two conclusions can be put forward. On the one hand, it can be said that the price of the S & P 500 index (S & P500) is generally marked by an alternation between downward and upward trends and is therefore non-stationary. On the other hand, the first difference of our variable (DS & P500) shows a tendency to stationarity. In order to carry out the study of the stationarity of the studied series, we use the unit root tests, namely the ADF test, the Phillips-Perron test and the KPSS test.
Table 1
Results of unit root tests

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF</th>
<th>Test de Phillips-Perron</th>
<th>Test de KPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>t-Stat</td>
<td>Prob</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>[1]</td>
<td>0.79</td>
<td>0.8836</td>
</tr>
<tr>
<td>DS&amp;P500</td>
<td>[1]</td>
<td>12.75*</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

With: * denotes the acceptance of the absence of unit root at the 5% threshold;

[3]: the model with trend and constant;

[2]: the model without trend and with constant;

[1]: model without trend and constant.

These three selected tests converge towards the same results. Indeed, our variable (the price of index S & P500) is non-stationary in level, whereas it is in first difference. Consequently, the series of the S & P500 index is integrated in order one, I(1). To analyze a stationary stochastic process, econometricians use multiple models. The ARIMA model category (AutoRegressive Integrated Moving Average), known as the Box-Jenkins models (Box and Jenkins 1978), which focuses on the analysis of probabilistic or stochastic properties of time series and which, unlike regression models explaining variable \( Y_t \) by k regressors \( (X_1;X_2;X_3; \ldots;X_k) \), allows \( Y_t \) to be explained in the past, or the offset values, by itself and by the terms of stochastic errors. Generally, the financial time series is integrated of order 1, I(1). If for financial \( Y_t \) series I(1), \( DY_t=Y_t–Y_{t-1} \) can be modeled as an ARMA(1,1), where there is an autoregressive term and a term moving average. The ARMA is a short memory process in the sense that the effect of a shock at a given moment is not sustainable and does not affect the future development of chronic. The long memory process, but not infinite, where the effect of a shock lasting consequences for future values of the series but it will regain its natural equilibrium level, is formalized by a fractional ARIMA process noted ARFIMA (AutoRegressive Fractionally Integrated Moving Average) when the degree of differentiation is not an integer. This process was extended to seasonal event and noted SARFIMA process (Seasonal AutoRegressive Fractionally Integrated Moving Average) (Ray 1993; Porter-Hudak 1990). ARIMA models univariate linked to a single time series, can be extended to ARIMA multivariate models, named VAR models (Vector AutoRegression), where each variable is explained by its lagged values, or past, and lagged values of all other endogenous variables in the model. However, for financial time series, as the price of a stock, there are periods of high volatility and periods of relative calm. In other words, the first difference from a financial time series, which is stationary, has wide variations, or volatility, and thus has a variance (moment of order 2 of the
process) which changes over time. This nonlinearity of the variance cannot be modeled by the linear ARIMA model. To model this variable variance, econometricians use the ARCH model (AutoRegressive Conditional Heteroscedasticity) (Engle 1982) and its extensions GARCH (Generalized Conditional AutoRegressive Heteroscedasticity) (Bollerslev 1986), IGARCH (Integrated Generalized Conditional AutoRegressive Heteroscedasticity) EGARCH (Exponential Generalized Conditional AutoRegressive Heteroscedasticity) TARCH (Threshold AutoRegressive Conditional Heteroscedasticity), TGARCH (Threshold Generalized Conditional AutoRegressive Heteroscedasticity)... Similarly, the non-linearity may also relate to the sample average (moment of order 1). In this case, it is essential to use either non-linear extensions of ARMA model, namely: the bilinear model (Granger and Andersen 1978), which is linear with the term autoregressive and the term moving average but is not linear with these two terms taken together, that is to say in relation to their product, the TAR model (Threshold AutoRegressive model) (Tiao and Tsay 1994), which is a model assuming the plan existence of several dynamic for the same series (several schemes) and seeking to specify a transition mechanism from one regime to another. This mechanism can be considered either as exogenous transition mechanism governed by a string of Markov process: this is called Markov Switching Model (Hamilton 1989), or as an endogenous mechanism of transition where the transition function depends on the dependent variable and a threshold: it is called model threshold or TAR model that belongs next to the model family the very definition of the transition function, namely: TAR model, SETAR model (Self Exciting threshold AutoRegressive model) (Tersvirta and Anderson (1992)), EXPAR model (EXPonential AutoRegressive model)...

3. Speculative Bubbles: Formation, Burst and Explanatory Models

3.1. Definition of the Speculative Bubble, its Formation and its Burst

According to Boucher and Raymond (2009), the bubble is a speculative craze phenomenon for an asset, often but not necessarily financial. According to Le Page (2003), Charles Kindleberger has proposed in his famous book "Manias, Panics and Crashes" a list of assets that have been the focus of speculative euphoria. Of these assets, we find gold, real estate (hotels, condominiums, office buildings ...), the building lots, currencies, emerging industries (in the US), mergers and acquisitions, foreign direct investment (by US companies in the sixties and by US companies in 1980), the term investments on commodities and raw materials, purchase and sale options, companies entering the stock market. Furthermore, drawing from
Galbraith (1992)\(^1\), the famous speculative episodes are: the craze for tulip bulbs in Holland between 1634 and 1637, the bubble on the shares of the Company of Mississippi in 1720, the bubble on the shares of the South Sea Company in 1720, those on land, real estate and later on railways (crisis in 1873), the craze for the values of the Roaring twenties (Radio Corporation of America, Ford, ...) that preceded the crash in 1929, rising 243% of the stocks listed in the New York stock Exchange between 1982 and 1987 as well as the Internet bubble between 1998 and 2000. To which is added the housing bubble between 1995 and 2006 and the financial bubble of Chinese equity markets between 2014 and 2015. Speculative bubbles are intended speculation in the rising price of an asset on a phase of appreciation durable enough and disconnected from the real economy, the actual value may deviate upward from that fundamental. They are characterized by a long period of increase followed by a deflation that reduces prices to their fundamental values. Their training and then burst depend on price expectations. According to Boucher and Raymond (2009), the bubble is characterized by a growing gap between the value of the asset market and its fundamental value estimated from its economic fundamentals outside of self-fulfilling expectations. However, a sudden drop in prices after a period of growth, also called: a crash, a price correction, readjustment, ..., reflects the bursting of a speculative bubble. Indeed, this bubble shows its face after taking a unanimous awareness of the overvaluation and therefore its explosion caused by massive sales orders.

According to its size, the bursting of a speculative bubble may take the form of a crash that exposed a crisis confined to the affected market; and, secondly, a global financial crash that causes a systemic crisis.

3.2. Explanatory Models of the Speculative Bubble

3.2.1. Speculative Bubbles Sound

According to the theory of rational bubbles, as investor expectations are validated by the subsequent evolution of their price or forecasts indicate that the latter will remain increasing, the investor interest to bet on its continued rise. These self-fulfilling expectations on the basis of information available fall into the category of rational expectations. While a bet too soon on a downward correction of prices may lead to shortfalls as the bubble grows. According to

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Tirole (1982, 1985), a rational bubble exists only where it believes a rate that does not exceed the growth rate of the economy in the long term. Similarly, assets whose price is limited, even with terminal condition, such as bonds, are not subject to rational speculative bubbles. Indeed, on the day immediately preceding the redemption date where the price is fixed, no speculator will not want to bet on the continued overvaluation. Result, the bubble associated with these assets with terminal condition ceases during the period preceding the redemption date. Moreover, according to Le Page (2003), for a bubble could develop, it is necessary that the horizon of the asset is infinite and without terminal condition. In fact, besides the above explanation on the terminal condition, if the horizon is infinite, the speculator can always hope resell the assets and speculative phenomena can develop. Moreover, rational bubble can never appear after some time of the creation of assets. Nevertheless, it must be present from the issue of the asset with an inflated price.

Finally, according to Boucher and Raymond (2009), in the second generation models, the bubbles are likely "congenital" because, even after their blow-ups, they will never disappear completely to be able to grow again.

3.2.2. Imperfect Information, Mimicry and Speculative Bubble

Assuming the perfect homogeneity of forecasts since the information is the same for all investors, the rational bubble theory does not explain the coordination of expectations to form a bubble. Moreover, according to the theory of mimicry, the available information is distributed asymmetrically between investors, because of its diverse modes of access, information concealment strategies, limited computing capabilities, etc. This theory then emphasizes the asymmetry of information to explain the herd behavior among investors.

For example, strategies "momentum", which are to follow the market trend, leading investors who can ignore their own information, to sell when the price drops or buy when it goes up and thus form a bubble. Thus, mimicry can be defined as a set of related individual behavior and not independent or imitations of behavior that can characterize the economic agents in the financial markets. It happens when an agent imitates the decision of one or more other agents, without giving proper weight to its own information, and can therefore modify its own decision. This theoretical current explains the mimetic behavior not three reasons: First, given the asymmetric nature of information and the possibility of detention of inside information by the other participants, the investor's interest to follow the market. This is called, for the record,

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in the literature a "waterfall model". Second, fund managers in the financial markets, who will not show results below those of their competitors, are encouraged to follow the decisions of others in order to maintain their market share and save thus their level of pay in the principal-agent models. According to Christophe Boucher and Raymond (2009), in the UCITS market (undertakings for the collective investment in transferable securities) in the principal-agent relationship that binds the fund manager to his employer (client of funds), it can be optimal for the latter, in a situation of moral hazard (uncertainty on the supplied effort) or adverse selection (uncertainty on skills), to establish a contract where the manager's remuneration is linked to its relative performance.

Result, fund managers prefer to follow a reference group or a benchmark. This management, benchmarking, is widespread and contributes to the standardization of practices. Third, according to the theory of self-referential mimicry of André Orléan (1989), investors are no longer based on an external reference to the market as the fundamental value but on data produced by the market itself, namely: majority opinion of investors, especially when payment receives one among them by adopting an action increases with the number of investors who adopt this same action (payment externalities). The example of the "beauty contest" advanced by John Maynard Keynes in his famous work The General Theory of Employment, Interest and Money (1936), is the best illustration of this reason. In this contest, the winner was the one who was able to choose the six pretty faces from a hundred photographs, approaching the highest average selection by all participants. However, in a context of asymmetric information, an asset may have different core values according to different forecasts of future dividends formed by speculators. In this context, Allen, Morris and Postlewaite (1993) propose two definitions of bubbles, namely: the "bubble in the weak sense" (expected bubble), which exists when the value of the asset exceeds all hoped fundamental values by speculators, and the "bubble in the strong sense" (strong bubble), which exists when no realization of dividends cannot justify such a high price. First, a "bubble in the strong sense" exists only when "bubble in the weak sense" exists. On the other hand, it exists in equilibrium when each speculator thinks that others are not aware of the "strong bubble." That is to say, the theory of mimicry, unlike the rational bubble theory with perfect information, which considers the existence of bubbles as a "common knowledge" considers the existence like bubbles a "mutual knowledge" shared by all agents without knowing.

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3 See De Boissieu, Christian. 2009, op.cit., p.82.
3.2.3. The Behavioral and Bubbles Limited to Arbitration

According to Boucher and Raymond (2008), in addition to rational investors, the approach applying behavioral models highlights the existence of ignorant investors (noise traders) buying and selling of assets on the basis of erroneous beliefs. On one hand, according to the theory of efficient markets, the presence of these ignorant investor has no impact on asset prices, as rational investors arbitrate when they observe any difference between the price observed and the fundamental value, that is to say, they sell the overvalued securities and they buy those undervalued. On the other, according to the theory of behavioral finance, the arbitration rational investors remains risky and limited, and cannot bring the prices to their fundamental values, and the irrationality ignorant investors may thus have a substantial and lasting impact on prices. In order to clarify this conclusion, the approach of behavioral finance four reasons. In the first place, according to De Long et al (1990), arbitration risky turns out when there is no close substitute to evil valued title or when the entire market is poorly valued. Second, massive purchases of ignorant investors can increase the short-term overvaluation. According to Shleifer and Vishny (1997), this short-term growth may be overvalued as generated by rational investors such as fund managers and Hedge funds which sometimes are asked to delay their operations to arbitration counter the impact of even temporary losses on the sales of their funds. Third, arbitration may induce some substantial transactions costs such as commissions, spreads bid-ask or the cost of finding information on the valuation of companies. Finally, rational investors find it difficult to synchronize their actions to eliminate the overvaluation, because each of them can easily anticipate when other investors will sell the assets overvalued and thus thwart the bubble. They can choose to "play" the bubble rather than trying to thwart.

4. The Markov Regime Switching Model

4.1. Presentation of the Model

Hamilton (1989) shows that the first difference of the observed financial series, which are generally non-stationary in level, following a nonlinear stationary process. Thus seeking to model the process of financial series as a process Markov-switching schemes, it determines each plan at time t by an unobservable variable denoted $S_t$. Generated by a Markov process of order 1, where the current $S_t$ regime depends only on the $S_{t-1}$ previous regime, this $S_t$ variable takes two possible states: the state of stability "0" and the state of crisis "1". It has the following transition probabilities: $\Pr(S_t=1/S_{t-1}=1)=P_{11}$ and $\Pr(S_t=0/S_{t-1}=0)=P_{00}$.

Its stochastic process is then represented by the matrix constant transition probabilities:
\[ \Theta = \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{pmatrix} \]  

(1)

Where: \( P_{ij} = \Pr(S_t = j / S_{t-1} = i) \) is the probability of moving from state \( i \) to state \( j \) and \( \sum_{j=0}^{1} P_{ij} = 1 \) for \( i \in \{0; 1\} \); that is to say: \( P_{01} = 1 - P_{00} \) and \( P_{10} = 1 - P_{11} \). The transition probabilities will take the following form logistics: \( P_{00} = \frac{\exp(P_0)}{1 + \exp(P_0)} \) and \( P_{11} = \frac{\exp(q_0)}{1 + \exp(q_0)} \).

Building on the model of Hamilton (1989), where only the average of the studied series can change from one plan to another, we will present a simple two-state model with a specification autoregressive of order 1\(^4\). This model is written as follows:

\[ y_t - \mu_{St} = \beta(y_{t-1} - \mu_{S_{t-1}}) + \epsilon_t \]  

(2)

With \( \epsilon_t \sim N(0; \sigma^2) \) and \( S_t \in \{0; 1\} \)

In this model\(^5\), there are two distributions with two different means governing the series \( y_t \). Is designated by the state \( S_t = 1 \) the crisis regime and the state \( S_t = 0 \) the stability regime. Like the Hamilton model (1989), it was assumed that the variance of the error term is constant between the two regimes.

### 4.2. Estimation of the Markov Regime Switching Model

The parameters of the equation (2) and the transition probabilities of the equation (1) are jointly estimated by the method of maximum likelihood.

In order to build the likelihood function of model (2), we adopt the approach proposed by Hamilton (1994) in the following steps:

i) The conditional density function of \( y_t \) knowing \( S_t \) is:

\[ f(y_t | S_t = 1; \theta) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left[ -\frac{(y_t - \mu_{s_1} - \beta(y_{t-1} - \mu_{s_{t-1}}))^2}{2\sigma^2} \right] \text{ for } j=0; 1. \]  

(3)

ii) The unconditional probability that one is in state \( j \) at time \( t \), \( S_t = j \) is:

\[ P_r(S_t = j ; \theta) = \pi_j \text{ for } j=0; 1. \]  

(4)

Where: \( \theta = (\mu_0; \mu_1; \beta; \sigma^2; \pi_0; \pi_1)' \) is the vector of parameters to be estimated.

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\(^4\) Hamilton (1989) used a two-state model with a specification autoregressive of order 4.

\(^5\) Which can also be written as follows: \( Y_t = \mu_0(1-S_t) + \mu_1 S_t + \beta (Y_{t-1} - \mu_0(1-S_{t-1}) - \mu_1 S_{t-1}) + \epsilon_t \)
According to Hamilton (1989, 1994), inferring the unobserved variable (St) is performed by an iterative algorithm, such as: the Expectation Maximization "EM" algorithm. To start the iteration process, Hamilton (1994) shows that we can use the following initial unconditional probabilities: 

$$\pi_0 = \frac{1-P_{11}}{2-P_{11}+P_{00}} \text{ and } \pi_1 = \frac{1-P_{00}}{2-P_{11}+P_{00}}.$$  

iii) In light of Bayes theorem, we deduce the joint density function of $y_t$ and state $S_t$.

$$P_r(y_t, S_t; \theta) = f(y_t | S_t = j; \theta)\pi_j \text{ for } j=0; 1. \quad (5)$$

$$P_r(y_t, S_t; \theta) = \frac{\pi_j}{\sqrt{2\pi}} \exp\left[\frac{-(y_t-\mu_{st}-\beta(y_{t-1}-\mu_{st-1}))^2}{2\sigma^2}\right] \text{ for } j=0; 1. \quad (6)$$

iv) The unconditional density function of $y_t$ for all possible states of $S_t$ is:

$$f(y_t; \theta) = \sum_{j=0}^{1} f(y_t, S_t = j; \theta) \text{ for } j=0; 1. \quad (7)$$

$$f(y_t; \theta) = Pr(y_t, S_t = 0; \theta) + Pr(y_t, S_t = 1; \theta) \text{ for } j=0; 1. \quad (8)$$

$$f(y_t; \theta) = \frac{\pi_0}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(y_t-\mu_0-\beta(y_{t-1}-\mu_0))^2}{2\sigma^2}\right] + \frac{\pi_1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(y_t-\mu_1-\beta(y_{t-1}-\mu_1))^2}{2\sigma^2}\right] \quad (9)$$

v) The unconditional probability density function is used above and gives the following likelihood function:

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t; \theta) \text{ under the constraints } \pi_0 + \pi_1 = 1 \text{ and } \pi_j \geq 0 \text{ for } j=0; 1 \quad (10)$$

To estimate the parameter $\theta$, we can use the expectation-maximization algorithm. After estimating the parameter vector, it is possible to use the equations (6) and (9) to calculate the probability of each conditional plan in $y_t$ observation date $t$:

$$P_r(S_t = 1 | y_t; \theta) = \frac{P_r(y_t, S_t = 1; \theta)}{f(y_t; \theta)} = \frac{\pi_1 f(y_t | S_t = 1; \theta)}{f(y_t; \theta)} \quad (11)$$

This is, given the observed data at time $t$, the probability that the plan at this point is the regime j. This equation (11) represents the probability that the regime of observation $t$ is the regime j, given the observed data at this time.

5. Application of the Markov Regime Switching Model

5.1. First Difference of the Courses of the S&P 500 During the Period 2000-2015

Inspired by Hamilton (1989), we will estimate a switching model in both states and with a specification autoregressive of order 1, the first difference in the monthly price of the S&P 500 during the period running from December 1999 to December 2015.
Table 2
Estimation of the Markov regime switching model

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated coefficient of Markov regime switching model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_0)</td>
<td>21.80178 [0.0000]</td>
</tr>
<tr>
<td>(\mu_1)</td>
<td>-75.87128 [0.0000]</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.198561 [0.0068]</td>
</tr>
<tr>
<td>(\log(\sigma))</td>
<td>3.696109 [0.0000]</td>
</tr>
</tbody>
</table>

Where: [.] is critical probability;

\[
DSANP500_t = \mu_0 (1 - S_t) + \mu_1 S_t + \beta (DSANP500_{t-1} - \mu_0 S_{t-1}) + \epsilon_t
\]

with: \(\epsilon_t \sim N(0;\sigma^2)\); \(S_t \in \{0; 1\}\) where the state \(S_t = 1\) is the crisis regime and the state \(S_t = 0\) is the stability regime; \(P_{ij} = \Pr(S_t = j/S_{t-1} = i)\) is the probability of moving from state \(i\) to state \(j\) and \(\sum_{j=0}^1 P_{ij} = 1\) for \(i \in \{0; 1\}\); that to say \(P_{10} = 1 - P_{11}\); and \(P_{00} = 0.914619\) and \(P_{10} = 0.387076\).

To validate this model, we verified that the residue is a white noise. Indeed, the Q-statistic of Ljung-Box is equal to \(Q(36) = 44.959\), with a critical probability equal to 0.145. The null hypothesis is accepted and the residue is a white noise.

After the analysis of this model, the first outcome is that the estimated parameters are significant at the 5% statistical; and, secondly, there are changes in different schemes in the first difference in the price of the S & P index of 500. In fact, there are two states: an optimistic or stable state, positive average equal to 21.80178, and another pessimistic or crisis, negative mean of \(-75.87128\). In addition, the state of stability, which has a transition probability of \(P_{00} = 0.914619\), is more persistent compared to the crisis, which has a transition probability of \(P_{11} = 0.612924\). Moreover, the unconditional probabilities of the state of stability and the crisis, which are equal to \(\pi_0 = \frac{1 - P_{11}}{2P_{11} - P_{00}} = 0.81928\), \(\pi_1 = \frac{1 - P_{00}}{2P_{11} - P_{00}} = 0.18072\), respectively and indicate that, for a given sample of the first difference of the index of S&P 500 close to 18.072% of the comments should be in a state of crisis. In other words, the state of tranquility dominates most of the time instead of Wall Street Stock Exchange. It was also found that the conditional expected duration in the state of crisis equals \(\frac{1}{1 - P_{11}} = 2.58347\) months. That is to say, one can expect, on average, a high volatility period lasts about two and a half month. It should also be noted that instead of Wall Street Stock Exchange has a chance to move from the state of crisis in \(t-1\) to that stability does, estimated
at $P_{10} = 0.387076$, greater than that of moving from state of stability $t-1$ than in summer crisis, estimated at $P_{01} = 0.085381$.

5.2. Smoothing Probability Corresponding to the State of Crisis During the Period 2000-2015

The following figure shows the evolution of the probabilities of being in crisis phase, called "smoothing probability", for Wall Street Stock Exchange.

Where:

$$P_t(S_t = 1|DS&P500_t; \theta)$$

is the smoothed probability of the crisis regime; and $\theta = (\mu_0; \mu_1; \beta; \sigma^2; \pi_0; \pi_1)'$ is the vector of parameters to be estimated.

Given the evolution of the probability of being in crisis phase, called "smoothing probability" to the Wall Street Stock Exchange, shows that this probability is greater than or equal to 0.6 for very volatile periods the first difference of the index of S & P 500, after the crisis of TMT in 2000-2001, after the financial crisis in 2007-2008, after the European debt crisis in 2010 and after the financial crisis in China in 2015. Consequently, all financial crises of 2000-2015 were detected by the Markov switching model.
6. Conclusion

Our model has allowed the detection of three major financial crises in 2001, 2008 and 2015, respectively, after the bursting of the Internet bubble (1998-2000), the bursting of the housing bubble (1995-2006) and the bursting of the Chinese financial bubble in 2015. Indeed, the probability of being in crisis phase (probability smoothing) is greater than 0.6 after the crisis of TMT between 2000 and 2001, after the financial crisis between 2007 and 2008, after the European debt crisis in 2010 and after the Chinese financial crisis in 2015. In addition, the state of stability, which has a transition probability of $P_{00} = 0.914619$, is more persistent compared to the crisis, which has a transition probability of $P_{11} = 0.612924$. Moreover, the unconditional probabilities of the state of stability and the crisis, which are equal to $\pi_0 = 0.81928$, $\pi_1 = 0.18072$ respectively and indicate that, for a given sample of the first difference of the index of S & P 500 close to 18.072 % of the comments should be in a state of crisis. In other words, the state of tranquility dominates most of the time instead of Wall Street Stock Exchange. It was also found that the conditional expected duration in the state of crisis equals 2.58347 months.

That is to say, one can expect, on average, a high volatility period lasts about two and a half month. It should also be noted that Wall Street Stock Exchange has a chance to move from the state of crisis in $t-1$ to the state of stability, estimated at $P_{10} = 0.387076$, greater than the chance of moving from the state of stability in $t-1$ to the state of crisis, estimated at $P_{01} = 0.085381$.

Certainly, knowledge of financial shocks that may occur allows taking proactive measures to ensure financial stability. However, their forecasts are very delicate. Evidenced, questioning "why no one did not see this crisis coming?", and sending in November 2008 by Queen Elizabeth to professors from the famous University London School of Economics (Ben Hammouda et al. 2011). In addition, Isaac Newton, part of investors ruined when the bursting of the bubble of the Companion of the South Seas in 1720, confesses: "I can measure the motions of heavenly bodies, but I cannot measure human nonsense" (Lacoste 2009).
References


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