ABSTRACT
The aim of this article is to compare the GARCH (Generalised Auto-Regressive Conditional Heteroskedasticity) family models of GARCH (1.1), GJR-GARCH, PGARCH, EGARCH, and IGARCH, to the EWMA (Exponentially Weighted Moving Average) model in the hope of finding the best model to forecast the volatility of the Moroccan stock market index MADEX. It proposes an empirical approach based on the assessment and measurement of the forecasting models based on well-known forecasts error metrics such as MRSE, RAE, and TIC. The data used in this study consists of a series of daily returns covering the period between 01/04/1993 and 30/08/2016. The results of the study confirm the superiority of the GARCH family models in terms of volatility forecasting since the asymmetric model of IGARCH, following a normal error distribution, seems to yield the best forecasting performance results thus outperforming the EWMA model. Its conclusions could be of a particular interest to hedge funds and portfolio managers along with investors acting in the Moroccan context.

Keywords
Volatility Forecasting
Volatility Modeling
Stylized Facts
GARCH Family Models
EWMA

*Corresponding Author:
eljebari.ouael@gmail.com

Author(s) retain copyright of the submitted paper (Please view the Copyright Notice of JIFM).
1. Introduction

The modeling of volatility in financial markets has become a major focus in the world of finance, due, of course, to the growing and crucial role exhibited by volatility in financial markets. Today, volatility is a central feature of contemporary financial markets, and its uses are as broad and diversified as they can be. It is currently an essential component in the process of value-at-risk measuring, portfolio management, options pricing, along with many other financial decision-making practices.

Volatility is presently a paramount variable in the valuation of derivatives; it allows the construction of stock market indices serving as a proxy for measuring the levels of uncertainty among investors and actors in financial markets.

With the 1996 Basel Accord, volatility forecasting has officially become a mandatory task for financial institutions across the globe, therefore, demonstrating the importance of volatility in the international financial sphere.

Changing levels of volatility in financial markets could have major repercussions on the global economy, given the way and the extent to which the latter reacts to political and economic shocks, as well as its exponential relationship with the arrival of News, especially when it comes to bad news.

Being aware of the increasing role of volatility in the practice of risk management, we have decided in this article, to focus on the issue of determining and inquiring for the best model to predict the volatility of the Moroccan stock market index MADEX.

The motivation behind this article prevails, in addition to the previously exposed reasons, in the relative lack of academic research on the subject for the Moroccan context, as well as the partial outreach of previous studies.

2. Literature Review

The forecasting of volatility, along with, the comparison of the out-of-sample forecast performance of the different models, is a booming subject, and several researchers have begun to work on this topic. For instance, Akgiray (1989) in his study discovered a certain superiority of GARCH models in regards to EWMA (exponentially weighted moving averages), ARCH (AutoRegressive Conditional Heteroskedasticity), as well as the historical average model, in predicting the monthly volatility of the US stock index.
A similar conclusion was obtained by West and Cho (1994) using the one-step-ahead forecast of the dollar exchange rate.

Despite the fact that there exist numerous techniques for modeling volatility in the financial markets, the literature review concludes that the essential elements of the studies are carried out using models from the GARCH family. This is largely due to their ability to account for all the stylized facts often observed on financial markets, among which we can cite:

- Squared returns are positively correlated, meaning that significant changes in the price of a financial asset at time $t$ will imply a significant change in price levels at time $t + 1$.
- The series of financial asset prices are marked by an excess of kurtosis, the equivalent of fat tails. Fama (1965) and Mandelbrot (1963) were the first to point out this non-normality of the financial series.
- Volatility tends to cluster, meaning that periods of high volatility are followed by periods of high volatility, and periods of low volatility are followed by periods of low volatility.
- Leverage: the evolution of financial prices is negatively correlated with volatility. Black (1976) explains that the more than proportional change caused by price volatility can only be explained by leverage. More empirical evidence on this stylized fact is shown in the article by Engle and Ng (1993).
- Long memory: volatility is very persistent especially in the case of high-frequency data, there is even evidence of unit root in the process of conditional volatility.
- Correlation of volatility: the observation of several financial assets and more specifically the exchange rates shows the existence of the correlation of volatility between one currency and another.
- The mean-reverting property: when volatility is disrupted, it tends to return to its mean, which itself may be altered over time.
- Risk premium: the riskiest assets with large variances are the most profitable assets.
- Uncertainty in macroeconomic aggregates implies volatility in financial markets.
- Pagan and Schwert (1990) compared the GARCH, EGARCH, Markov Regime Switching model and three other nonparametric models in an attempt to predict the monthly volatility of US stock market returns. According to this paper, the GARCH model closely followed by EGARCH functioned in a moderate manner while the rest of the models performed poorly.
Franses and Van Dijk (1996) compared three models of the GARCH family: GARCH, QGARCH, and GJR-GARCH, to forecast the weekly volatility of several European stock indices. They identified by the end of this study that the nonlinear models could not surpass the standard GARCH model. Brailford and Faff (1996) found that the GJR-GARCH coupled with GARCH were slightly superior to several simple models to predict the monthly volatility of the Australian stock market.

Engle and Patton (2001) were able to prove the ability of GARCH models to account for the stylized facts observed on the volatility of the Dow Jones stock index.

Lupu and Lupu (2007), discovered by working on a daily series covering the period between 03/01/2002 to 17/11/2005, that the EGARCH model is the best to express the volatility of the stock index of Romania BET-C. Miron and Tudor (2010) worked on several types of asymmetric GARCH models (EGARCH, PGARCH, and TGARCH), using stock indices from the US and Romania covering the period between 2002 and 2010. They were able to demonstrate that the estimation of the volatility resulting from the application of the EGARCH model is much more reliable than the estimates made by the other models.

GARCH models represent a generalization of the ARCH models (autoregressive conditional heteroscedasticity), this type of models was developed for the first time by Engel (1982) as an ARCH (q) where conditional volatility was a function of q delays of Past squared yields.

The models of the ARCH family have been extensively studied, and in particular, we can cite the works of Bollershev, Chou, and Kroner (1992), along with Bollerslev, Engle, and Nelson (1994). The GARCH models were an extension to the ARCH family models and were developed by Bollerslev (1986) and Taylor (1986). The main contribution of these models is to allow, in addition to the term ARCH (q), another term GARCH (p) to represents the delays of the conditional volatility $h_t$ itself.

Given the great success of these models, several extensions have been developed to try to perfect this theoretical framework and make it extra efficient. Among these extensions, figures the exponential EGARCH or GARCH Nelson (1991) where the conditional volatility is specified in the logarithmic form, which indicates that there is no need to impose estimation constraints to avoid the problem of Negative variance.

This property allows the user to take into account the stylized fact that negative shocks imply a greater variation of volatility compared to positive ones. Another non-symmetric model with characteristics close to EGARCH is the TGARCH also called GJR-GARCH developed
by respectively Zakoian (1994) and Glosten, Jagannatan, and Runkel (1993). The main difference between TGARCH and EGARCH is that the TGARCH applies the conditional standard deviation instead of the conditional variance.

While shocks in the volatility series tend to exhibit long memories, and as a result, tend to impact future volatility for a long horizon, the IGARCH or the Integrated GARCH was proposed by Engel and Bollerslev (1986) To capture this stylized fact, as well as to make conditional volatility infinite, and shocks permanent.

Similarly, Ding et al (1993) proposed the PGARCH (Power GARCH) model, which came to provide another method for modeling the long memory property in volatility. An excellent review of volatility prediction models can be found at Poon and Granger (2003).

3. Data and Methodology

The data used in this article consists of 5839 daily price observations of the MADEX Moroccan stock index, spanning the period from 01/04/1993 to 15/08/2016. The time series of MADEX prices were divided into two series, the first one covering the period from 01/04/1993 to 15/08/2015, or 5,592 observations, and was used to estimate our models of EWMA and those of the GARCH family, as well as to compute the descriptive statistics. However, the second series covering the period between 15/08/2015 and 15/08/2016, or 247 observations, was used to evaluate the out-of-sample forecast performance of each of our models. With this decomposition, we will be able to compare the "future" volatility forecasts while having values that have not been included in model estimation as a reference. Therefore, we will not be limited to the only in-sample observations.

At this stage, it should be noted that our choice of models of the GARCH family is motivated by their great ability to capture the stylized facts often observed in the international financial markets. Similarly, our choice to opt for EWMA is explained by its non-return to average property.

To the best of our knowledge, this article represents the first attempt to compare and study several models in order to capture and model the features of the conditional volatility of the Moroccan stock exchange market, yielding consequently, quality forecasts which can be deemed necessary for Moroccan risk managers.

In terms of methodology, we have chosen to work with the five main extensions of the GARCH family models, which are: GARCH (1.1), GJR-GARCH, EGARCH, PGARCH, and
IGARCH, in addition to EWMA model. The mathematical formulation of each of these models is set out as following.

**GARCH (Generalised AutoRegressive Conditional Heteroskedasticity)**

Bollerslev (1986) and Taylor (1986) developed the GARCH (p, q) model, allowing the conditional variance of the variable to be dependent on previous delays and capturing information and news contained in historical values of the variance. This model is presented as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$  \hspace{1cm} (1)

As the notation shows, the GARCH (p, q) model contains, in addition to the term GARCH (h_{t-1}) or delays in the conditional variance, an ARCH (u_{t-1}^2) squared. In the financial literature, the GARCH model (1.1) remains by far the most used model, hence our choice to use this type of model.

The notation of the GARCH model (1.1) is presented below:

$$h_t = \alpha_0 + \alpha_i u_{t-1}^2 + \beta_1 h_{t-1}$$  \hspace{1cm} (2)

This model has a non-negativity constraint of the coefficients $\alpha$ and $\beta$. So that the variance is always positive, and the coefficient $\alpha_0$ must be always greater than 1.

**GJR GARCH (Glosten Jagannathan et Runkle GARCH)**

The GJR GARCH model is a simple extension of the GARCH model by adding an additional term to account for the asymmetries observed in the financial markets Brooks (2008, p.405). Glosten, Jagannathan, and Runkle (1993) have developed this model to allow conditional volatility to have different reactions to past innovations based on their signs. This model is presented as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \gamma_i u_{t-i}^2 \alpha_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j}$$  \hspace{1cm} (3)
Where $d_{t-1}$ is a dummy variable:

$$
d_{t-1} = \begin{cases} 
1 & \text{if } u^2_{t-1} < 0 \text{ negative shocks} \\
0 & \text{if } u^2_{t-1} \geq 0 \text{ positive shocks} 
\end{cases}
$$

And $\gamma$ is the coefficient that measures the impact of news influx; the other parameters of the equation remain the same as those of the GARCH model.

In this model, the consequence of good news can be appreciated through $\alpha_i$, whereas the impact of negative shocks is felt by $\alpha + \gamma$. Moreover, if $\gamma \neq 0$, the impact of news occurrence is set to be asymmetric, while, when $\gamma > 0$ then volatility is said to be marked by a leverage effect.

In order to be in line with the condition of non-negativity of the coefficients, it is necessary that $\alpha_0 > 0$, $\alpha_i > 0$, $\beta \geq 0$ and $\alpha_i + \gamma \geq 0$. However, the model could be still acceptable if $\gamma_i < 0$, Have $\alpha_i + \gamma \geq 0$. (Brooks, 2008).

**EGARCH (Exponential GARCH)**

For the GARCH Exponential or the EGARCH proposed by Nelson (1991), the conditional volatility specification is given by the following formula:

$$
\log(h_t) = \alpha_0 + \sum_{j=1}^{v} \beta_j \log(h_{t-j}) + \sum_{i=1}^{v} \alpha_i \frac{|u_{t-i}|}{\sqrt{h_{t-i}}} + \sum_{k=1}^{i} \gamma_k \frac{u_{t-k}}{\sqrt{h_{t-k}}} 
$$

(4)

Where $\log(h_t)$ represents the logarithm of conditional volatility, $\log(h_{t-1})$ is the logarithm of the first lag in conditional volatility, and $u_{t-i}$ stands for the term of the error at time $i$.

The use of the EGARCH model has the advantage to authorize the effects of information asymmetries to happen. In the EGARCH equation, $\gamma_k$ represents the leverage parameter used to capture the asymmetry. Thomas and Mitchell (2005).

The main contribution of this model lies in its capacity to accommodate negative shocks by allowing them to exhibit a greater impact on volatility in contrast to positive shocks.
**Power GARCH (p,d,q)**

This model was proposed by Ding et al. (1993) and has the advantage of being able to capture and model the long memory property often observed in the series of volatility. It is presented as follows:

\[
h_t^d = \alpha_0 + \alpha (|u_{t-1}| + \gamma u_{t-1})^d + \beta h_{t-1}^d
\]

(5)

Where \(d\) is a power term, \(u_{t-1}\) represents the first lag of the error term (term ARCH), and \(h_{t-1}\) is the first lag of the conditional volatility. The term of power denoted \(d\) captures the standard deviation when \(d = 1\) and captures the conditional variance when \(d = 2\). The asymmetry is counted by the term \(\gamma\). Chapman and Hall (2009).

**IGARCH (Integrated GARCH)**

The IGARCH models introduced by Engel and Bollerslev (1986) have the advantage of providing a statistical response to the problem of the presence of a unit root in the time series of volatility, which renders volatility shocks permanent. It is an integrated model of volatility. The model’s mathematical formula is presented below:

\[
h_t^2 = \alpha_0 + \sum_{i=0}^{p} \alpha_i u_{t-i}^2 + \sum_{i=0}^{q} \beta_i h_{t-i}^2
\]

(6)

IGARCH models are said to be volatile models because current information remains valid for forecasting volatility across long horizons.

If \(\alpha_0 = 0\), it can be said that the series is integrated into variance to the order \(d\). And when \(\alpha_0 > 0\) then the series is integrated into the order \(d\) with a trend. \(d\) is the number of first differences needed in order to render it stationary.

As far as error distributions are concerned, the GARCH model theory suggests three assumptions about the distribution of residuals. These three assumptions imply that the residuals of the GARCH regression may follow a normal law, a student law, or a generalized error distribution (GED). Although the vast majority of GARCH models are based on a normal distribution of residuals, the calibration and adequacy of the optimal model remain
closely dependent on these distributions. Thus our choice of sailing through the three distributions, in a way to emphasize the contribution of our study.

**EWMA (Exponentially Weighted Moving Average)**

The EWMA model is one of the oldest econometric models and has been mainly developed as a response to the weaknesses of the simple volatility and historical volatility models, which assign the same weight to the past observations.

In fact, the weight of recent information tends to be more important than that of old observations, and, this is what makes EWMA a very powerful model despite its relative simplicity. Unlike GARCH models, EWMA has the advantage of a non-return to average, which is considered by many researchers to be a weakness of GARCH models Ding and Meade (2010). It is for this reason that there is a fairly large amount of work that suggests EWMA's ability to surpass GARCH models in forecasting and modeling volatility. The EWMA can be presented as:

\[
\sigma^2_n(\text{EWMA}) = \lambda \sigma^2_{n-1} + (1 - \lambda) r^2_{n-1}
\]

Where: \(\sigma^2_n\) denotes volatility at time n, \(\sigma^2_{n-1}\) is the first volatility lag, \(r^2_{n-1}\) is the square of the returns of period n-1 and finally \(\lambda\), which is called the smoothing coefficient. Based on recommendations of RiskMetrics, the value of \(\lambda\) was specified at 0.94 when the frequency of observations is daily. The term \((1 - \lambda) r^2_{n-1}\) represents the response intensity of the variance to market news, while \(\lambda \sigma^2_{n-1}\) is used to capture persistence in volatility.

The followed approach for this empirical study is to start by first estimating the conditional volatility of the MADEX index by going through the different GARCH models and exploring different error distributions. Then selecting afterward, the best models in function of the significant Parameters as well as the information criteria of Akaike (AIC), Schwarz (BIC), and maximum likelihood. Once we have obtained the best GARCH models, which best expresses, the volatility of our index, we will compare the forecasting performance of these models with EWMA, using the following statistics: RMSE (root mean squared error), MAE (mean absolute error) and TIC (Theil inequality coefficient).
4. Empirical Results

In order to properly conduct our study and to be in compliance with the financial theory in relation to these subjects, we have transformed our raw data series into a series of logarithmic returns according to the following function (without adjusting for dividends):

\[
r = \log \left( \frac{x_t}{x_{t-1}} \right)
\]

(8)

Where \(X_t\) represents the price of the sector index at time \(t\), and \(X_{t-1}\) refers to the price of the sector index in \(t-1\).

Afterwards, we have applied the Augmented Dickey-Fuller unit root test (ADF) to study the stationarity of the return series. Similarly, we have used the White test for the purpose of testing the ARCH effect or the heteroscedasticity property of the errors, this test was conducted on the residuals series taken from the following mean model regression:

\[
r_t = c + r_{t-1} + \varepsilon
\]

(9)

\(r_t\) is being the return at moment \(t\), while \(r_{t-1}\) stands for returns at moment \(t-1\).

Table 1
The ADF test results

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF Value</th>
<th>t-stat 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns of MADEX (r)</td>
<td>-57,574***</td>
<td>-3.96</td>
</tr>
</tbody>
</table>

***: values statistically significant at the levels of risk of 1%, 5%, and 10%

Table 2
White’s heteroskedasticity test results

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>White’s statistic</th>
<th>Obs R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns of MADEX (r)</td>
<td>408,563***</td>
<td>632,12</td>
</tr>
</tbody>
</table>

***: values statistically significant at the levels of risk of 1%, 5%, and 10%

From the results presented in tables 1 and 2, we conclude that the newly created MADEX return series is a stationary series. Similarly, the statistical significance of the White test led to the rejection the null hypothesis of the homoscedasticity of errors, for the acceptance of the alternative hypothesis of the heteroscedasticity of errors. At this stage, one can safely proceed to the estimation of our models, as the conditions for ARCH and GARCH modeling are met.
Figure 1
Plots of the evolution of MADEX returns and prices

Table 3
Summary statistics of the MADEX returns

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000347</td>
</tr>
<tr>
<td>Median</td>
<td>0.000157</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0536490</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.050935</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.007561</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>-0.016580</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.459888</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>10560.00</td>
</tr>
<tr>
<td>Probability</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

The table of descriptive statistics of the MADEX returns series displays a significant difference between the maximum and minimum values, which is synonymous with high volatility in the series, also, the significant difference between the value of the standard deviation and the mean could only reinforce this finding. The kurtosis value being very large compared to the value of 3 suggests the presence of a fat tail to the right of the mean, hence the nonnormality of the series. This non-normality is confirmed by the Jarque-Bera test which is significantly different from zero, therefore the normality hypothesis of the series cannot be accepted.

The empirical results of the regressions will be presented hereafter in function of the error distributions. The AIC, BIC and maximum likelihood criteria are used to find the optimal
model so that AIC and BIC are minimized, and the maximum likelihood is maximized independently of the error distributions.

**Table 4**
Results of the regressions following a Gaussian error distribution

<table>
<thead>
<tr>
<th>Conditional volatility model</th>
<th>( C )</th>
<th>( ARCH(1) )</th>
<th>( GARCH(1) )</th>
<th>Leverage</th>
<th>AIC</th>
<th>BIC</th>
<th>Maximum likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1.1)</td>
<td>3.66E-06</td>
<td>0.280965</td>
<td>0.681808</td>
<td>-</td>
<td>-7.289434</td>
<td>-7.28467</td>
<td>20316.01</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>3.61E-06</td>
<td>0.032846</td>
<td>0.684658</td>
<td>0.26247</td>
<td>-7.289476</td>
<td>-7.2835</td>
<td>20317.13</td>
</tr>
<tr>
<td>EGARCH</td>
<td>-1.14839</td>
<td>0.410135</td>
<td>0.915581</td>
<td>-0.0187</td>
<td>-7.295353</td>
<td>-7.2891</td>
<td>20333.50</td>
</tr>
<tr>
<td>PGARCH(1.1.1)</td>
<td>0.000590</td>
<td>0.245080</td>
<td>0.736882</td>
<td>0.03746</td>
<td>-7.295313</td>
<td>-7.28936</td>
<td>20333.39</td>
</tr>
<tr>
<td>PGARCH(1.2.1)</td>
<td>0.000444</td>
<td>0.261248</td>
<td>0.750706</td>
<td>0.057984</td>
<td>-7.431567</td>
<td>-7.42443</td>
<td>20714.06</td>
</tr>
<tr>
<td>IGARCH</td>
<td>-</td>
<td>0.073089</td>
<td>0.926911</td>
<td>-</td>
<td>-7.229333</td>
<td>-7.22695</td>
<td>20146.54</td>
</tr>
</tbody>
</table>

(.): P values.

The first observation to be drawn from this table is that the majority of the parameters are significantly different from zero, which underlines the high validity of our models. The sum of the terms \( \alpha \) and \( \beta \) for the models GARCH, PGARCH and IGARCH are very close to 1, which is explained by a rather significant presence of persistence in the volatility of the MADEX index. However, the value of \( \alpha \) is rather less than that of \( \beta \), which means that the negative shocks on the conditional volatility of MADEX do not have a greater impact on volatility than those of positive shocks.

For the asymmetric GARCH models, half of the parameters \( \gamma \) are statistically different from zero, which implies that the volatility of the MADEX index is asymmetric, hence the existence of leverage effects. The parameter \( \gamma \) of the PGARCH (1.1.1) model being statistically significant and having a positive value suggests that the impact of positive shocks on the volatility of the MADEX index is greater than that of negative shocks.
For the Gaussian distribution, the best model of the conditional volatility of the MADEX index is EGARCH, which presents significant parameters and have the smallest AIC, BIC values while having the greater maximum likelihood value. This model is closely followed by the models of GARCH (1.1), IGARCH and PGARCH (1.1.1). So these are the models that will be evaluated later to test and compare their predictive performance. The other models are eliminated for having non-significant parameters.

Table 5  
Results of the regressions following a student error distribution

<table>
<thead>
<tr>
<th>Conditional volatility models</th>
<th>C</th>
<th>ARCH(1)</th>
<th>GARCH(1)</th>
<th>Leverage</th>
<th>AIC</th>
<th>BIC</th>
<th>Maximum of likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1.1)</td>
<td>3.05E-06</td>
<td>0.376660</td>
<td>0.561468</td>
<td>-</td>
<td>-7.417003</td>
<td>-7.41108</td>
<td>20672.48</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>3.01E-06</td>
<td>0.345563</td>
<td>0.564075</td>
<td>0.05938</td>
<td>-7.417147</td>
<td>-7.41001</td>
<td>20673.88</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.15)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EGARCH</td>
<td>-0.99884</td>
<td>0.471150</td>
<td>0.933200</td>
<td>-0.0223</td>
<td>-7.425259</td>
<td>-7.41813</td>
<td>20696.51</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.059)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGARCH(1.1.1)</td>
<td>0.00041</td>
<td>0.286550</td>
<td>0.748209</td>
<td>0.04001</td>
<td>-7.427746</td>
<td>-7.42061</td>
<td>20703.41</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.1708)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGARCH (1.2.1)</td>
<td>3.01E-06</td>
<td>0.374696</td>
<td>0.664039</td>
<td>0.03960</td>
<td>-7.712</td>
<td>-7.705</td>
<td>21563.21</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.1070)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IGARCH</td>
<td>-</td>
<td>0.116128</td>
<td>0.833872</td>
<td>-</td>
<td>-7.377404</td>
<td>-7.37383</td>
<td>20560.14</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

( ): P values.

Results obtained following a student error distribution are closely related to those of the normal distribution. For this errors distribution, we can clearly see that the model GARCH (1.1) is the best to capture and model conditional volatility of our index. The parameters γ being entirely not statistically significant imply the non-existence of leverage effects in conditional volatility of MADEX.

For the case of this distribution, the only models that will be kept for the final study are the models of GARCH (1.1) and IGARCH.
Table 6
Results of the regressions following a generalized error distribution

<table>
<thead>
<tr>
<th>Conditional volatility model</th>
<th>C</th>
<th>ARCH (1)</th>
<th>GARCH (1)</th>
<th>leverage</th>
<th>AIC</th>
<th>BIC</th>
<th>Maximum likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1.1)</td>
<td>2.95E-06</td>
<td>0.321129</td>
<td>0.675124</td>
<td>-</td>
<td>-7.42254</td>
<td>-7.41659</td>
<td>20687.90</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>2.90E-06</td>
<td>0.289996</td>
<td>0.678385</td>
<td>0.061638</td>
<td>-7.42283</td>
<td>-7.41570</td>
<td>20689.73</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.083)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EGARCH</td>
<td>-0.9963</td>
<td>0.422749</td>
<td>0.931471</td>
<td>-0.02703</td>
<td>-7.42954</td>
<td>-7.42240</td>
<td>20708.42</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.039)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGARCH (1.1.1)</td>
<td>0.000444</td>
<td>0.261248</td>
<td>0.750706</td>
<td>0.057984</td>
<td>-7.43156</td>
<td>-7.42443</td>
<td>20714.06</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.0777)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGARCH (1.2.1)</td>
<td>2.90E-06</td>
<td>0.320049</td>
<td>0.678411</td>
<td>0.048255</td>
<td>-7.42283</td>
<td>-7.41570</td>
<td>20689.73</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.07)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IGARCH</td>
<td>0.093894</td>
<td>0.906106</td>
<td>-7.38963</td>
<td>-7.38606</td>
<td>20594.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

( ): P values.

The EGARCH model represents the best technique to model the conditional volatility of MADEX in the case of the generalized error distribution. With the exception of the EGARCH model, all the parameters γ are not statistically significant, which implies the non-existence of the leverage effects and the asymmetry of the volatility of our stock index.

In addition to the EGARCH model, the GARCH (1.1) and IGARCH model will also be kept in order to compare their predictive performance in the final test.

At this stage and after studying and comparing the models of the GARCH family with the hope to find the best adjustments of these models, we will proceed to the last step which represents the aim and the object of this work. In this second step, we will present a comparison of the forecasting performance of the models GARCH (1.1), GJR-GARCH, EGARCH, PGARCH (1.1.1), PGARCH (1.2.1), IGARCH and, of course, the EWMA model.
<table>
<thead>
<tr>
<th>Volatility model</th>
<th>RMSE</th>
<th>MAE</th>
<th>TIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1.1)</td>
<td>0.005872</td>
<td>0.004327</td>
<td>0.954</td>
</tr>
<tr>
<td>GARCH(1.1)_t</td>
<td>0.005871</td>
<td>0.004329</td>
<td>0.961</td>
</tr>
<tr>
<td>GARCH(1.1)_GED</td>
<td>0.005870</td>
<td>0.004335</td>
<td>0.981</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.005875</td>
<td>0.004326</td>
<td>0.941</td>
</tr>
<tr>
<td>EGARCH_GED</td>
<td>0.005870</td>
<td>0.004336</td>
<td>0.983</td>
</tr>
<tr>
<td>PGARCH(1.1.1)</td>
<td>0.005873</td>
<td>0.004327</td>
<td>0.951</td>
</tr>
<tr>
<td><strong>IGARCH</strong></td>
<td><strong>0.005876</strong></td>
<td><strong>0.004326</strong></td>
<td><strong>0.939</strong></td>
</tr>
<tr>
<td>IGARCH_t</td>
<td>0.005872</td>
<td>0.004328</td>
<td>0.955</td>
</tr>
<tr>
<td>IGARCH_GED</td>
<td>0.005870</td>
<td>0.004334</td>
<td>0.979</td>
</tr>
<tr>
<td>EWMA</td>
<td>0.005882</td>
<td>0.004339</td>
<td>0.943</td>
</tr>
</tbody>
</table>

From table 7, we can observe that the ten presented models are very close to each other; nevertheless, the analysis of the RMSE, MAE and TIC statistics makes it possible to conclude that the best model to forecast the volatility of the MADEX index, is the IGARCH with a normal error distribution. This model, compared to the others, yields the best results, by presenting the best values in 2/3 of the forecasting error statistics adopted in this study. Therefore, we can say that the models of conditional volatility are better than those of the exponentially weighted volatility for the case of the MADEX index.

5. Conclusion

Today, the forecasting of volatility in the financial markets is a major subject. Studies aimed at this subject continue to multiply, proposing each time new techniques and new models. Across this article, we tried to look for the best model to predict and forecast the volatility of the MADEX index. In order to achieve this, we have used GARCH models, which are widely studied and analyzed, and whose performances are largely documented in the financial literature.

As for the EWMA model, it was added to our sample models thanks to the interesting number of studies that have proved its superiority to the GARCH models, and to its main property of non-return to average.

Among the results obtained at the end of this study, we found that the GARCH models triumphed in modeling and explaining in a rather satisfactory manner the volatility of the Moroccan stock index compared to the EWMA model that has, nevertheless, succeeded in generating Very close estimates to those of the GARCH family models.
As for the main result i.e. the best model to forecast the volatility of the Moroccan stock index, the statistics of measurement of forecasting errors have affirmed the IGARCH model with a Gaussian distribution of errors as a rightful winner, and, hence, the superiority of the GARCH models in comparison to the EWMA model. The results obtained in this study could basically have applications in the practice of management of financial risks as they could serve as an inspiration for other researchers’ eager to have a better understanding of the dynamics of volatility in the Moroccan financial market.

Bibliography


